

Information algebra system of soft sets

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Abstract

Information algebra is algebraic structure for local computation and inference. Given an initial universe set and a parameter set, we show that a soft set system over them is an information algebra. Moreover, in a soft set system, the family of all soft sets with a finite parameter subset can form a compact information algebra.

Keywords: soft set, complete lattice, information algebra, compact information algebra.

1. Introduction

The information algebra system introduced by Shenoy [1] was inspired by the formulation of some basic axioms of local computation and inference under uncertainty [2]. It gives a basic mathematical model for treating uncertainties in information. Related studies [3, 4, 5] showed that the framework of information algebras covers many instances from constraint systems, Bayesian networks, Dempster-Shafer belief functions to relational algebra, logic and etc. Considering about the feasibility of information processing with computer, Kohlas [3, 4] presented a special information algebra with approximation structure called compact information algebra recently.

On the other hand, Molodtsov [6] initiated a novel concept, which is called soft set, as a new mathematical tool for dealing with uncertainties [7]. In fact, a soft set is a parameterized family of subsets of a given universe set. The way of parameterization in problem solving makes soft set theory convenient and simple for application. Now it has been applied in several directions, such as operations research [8, 9], topology [10, 11, 12], universal algebra [13, 14, 15, 16], especially decision-making [17, 18, 19, 20, 21].

It is thus evident that information algebra theory and soft set theory are both theoretical research tools for dealing with non-deterministic phenomenon. To study relationships between them is necessary. In this paper, we are concerned about the problem that whether there exist the frameworks of information algebras or even compact information algebras in soft sets. By choosing some appropriate operators,

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we construct an information algebra system of soft sets over an initial universe set and a parameter set. Then we further prove that, in a soft set system, the family of soft sets with a finite parameter subset can form a compact information algebra. These conclusions obtained in this paper demonstrate that soft set systems are also the instances of information algebras.

2. Preliminaries

In this section, first, we present some basic definitions about soft sets and some notations in lattice theory.

Suppose that (L, \leq) is a partially ordered set and $A \subseteq L$. We write $\vee A$ and $\wedge A$ for the least upper bound and the greatest lower bound of A in L respectively if they exist.

Let L be a partially ordered set. If $a \vee b$ and $a \wedge b$ exist for all $a, b \in L$, then we call L a lattice. If $\vee A$ exists for every subset $A \subseteq L$, we call L a complete lattice. Clearly, a partially ordered set L is a complete lattice if, and only if, L has the bottom element and $\vee A$ exists for all nonempty subset $A \subseteq L$.

A set $A \subseteq L$ is said to be directed, if for all $a, b \in A$, there is a $c \in A$ such that $a, b \leq c$. For $a, b \in L$, we call a way-below b , in symbols $a \ll b$, if and only if for all directed subsets $X \subseteq L$, if $\vee X$ exists and $b \leq \vee X$, then there exists an $x \in X$ such that $a \leq x$.

Let U be an initial universe set and E be a set of parameters, which usually are initial attributes, characteristics, or properties of objects in the initial universe set. $\mathcal{P}(U)$ denotes the power set of U .

Definition 2.1. ([6]) A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow \mathcal{P}(U)$.

Therefore a soft set is a tuple which associates with a set of parameters and a mapping from the parameter set into the power set of an universe set. In other words, a soft set over U is a parameterized family of subsets of the universe U . For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A) [6].

Definition 2.2. ([7]) A soft set (F, A) over U is said to be a null soft set, if for all $e \in A$, $F(e) = \emptyset$. We write it by (\emptyset, A) .

A soft set (F, A) over U is said to be an absolute soft set denoted by \tilde{A} , if for all $e \in A$, $F(e) = U$.

Definition 2.3. ([8]) The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B; \\ G(e), & \text{if } e \in B - A; \\ F(e) \cap G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \sqcap_\varepsilon (G, B) = (H, C)$.

In this paper we adopt the concept of information algebra given by Kohlas from [3]. For a full introduction and these abundant examples of information algebras, please refer to [3, 4, 5].

Definition 2.4. ([3]) Let (D, \leq) be a lattice. Suppose there are three operations defined in the tuple (Φ, D) :

1. Labeling d : $\Phi \rightarrow D; \phi \mapsto d(\phi)$, where $d(\phi)$ is called the domain of ϕ . For an $s \in D$, let Φ_s denote the set of all valuations with domain s .
2. Combination \otimes : $\Phi \times \Phi \rightarrow \Phi; (\phi, \psi) \mapsto \phi \otimes \psi$,
3. Marginalization \downarrow : $\Phi \times D \rightarrow \Phi; (\phi, x) \mapsto \phi^{\downarrow x}$, for $x \leq d(\phi)$.

If the system (Φ, D) satisfies the following axioms, it is called an information algebra:

1. Semigroup: Φ is associative and commutative under combination. For all $s \in D$, there is a neutral element e_s with $d(e_s) = s$ such that for all $\phi \in \Phi$ with $d(\phi) = s$, $e_s \otimes \phi = \phi$.
2. Labeling: For $\phi, \psi \in \Phi$, $d(\phi \otimes \psi) = d(\phi) \vee d(\psi)$.
3. Marginalization: For $\phi \in \Phi, x \in D, x \leq d(\phi)$, $d(\phi^{\downarrow x}) = x$.
4. Transitivity: For $\phi \in \Phi$ and $x \leq y \leq d(\phi)$, $(\phi^{\downarrow y})^{\downarrow x} = \phi^{\downarrow x}$.
5. Combination: For $\phi, \psi \in \Phi$ with $d(\phi) = x, d(\psi) = y, (\phi \otimes \psi)^{\downarrow x} = \phi \otimes \psi^{\downarrow x \wedge y}$.
6. Stability: For $x, y \in D, x \leq y, e_y^{\downarrow x} = e_x$.
7. Idempotency: For $\phi \in \Phi$ and $x \in D, x \leq d(\phi)$, $\phi \otimes \phi^{\downarrow x} = \phi$.

The items putting forward in the definition of information algebra can be seen as the axiomatic presentations of some basic principles in local computation and inference. Studies have shown this algebraic structure covers many instances from belief functions, constraint systems, relational databases, and possibility theory to relational algebra and logic([5]). For example, each lattice L is a simply information algebra on a domain set L itself. The operations are defined as follows:

1. Labeling d : For $x \in L$, $d(x) = x$.
2. Combination \otimes : $x \otimes y = x \vee y$.
3. Projection \downarrow : If $x \leq y$, $x^{\downarrow y} = x \wedge y$.

For an information algebra (Φ, D) , we introduce a order relation as follows:

$$\psi \leq \phi, \text{ if } \psi \otimes \phi = \phi.$$

This order relation induced by the operation combination is a partial order on the set Φ , if (Φ, D) is an information algebra.

3. Information algebra of soft sets

In this section, with these operations of soft sets defined above, we will construct an information algebra of soft sets. Let U be an initial universe set and E be a set of parameters. $\mathcal{S}_{U,E}$ (or simply \mathcal{S} when this doesn't lead to confusions) denotes the set of all soft sets (F, A) over U , where $A \subseteq E$, that is,

$$\mathcal{S} = \{(F, A) : (F, A) \text{ is a soft set over } U, \text{ where } A \subseteq E\}.$$

Three operations are defined as follows:

1. Labeling d : For a soft set (F, A) , we define $d((F, A)) = A$.
2. Projection \downarrow : If $B \subseteq A$, we define $(F, A)^{\downarrow B}$ to be a soft set (G, B) such that for all $b \in B$, $G(b) = F(b)$.
3. Combination \otimes : For any two soft sets $(F, A), (G, B) \in \mathcal{S}$, we define

$$(F, A) \otimes (G, B) = (F, A) \sqcap_{\varepsilon} (G, B).$$

We call a quintuple $(\mathcal{S}, \mathcal{P}(E), d, \sqcap_{\varepsilon}, \downarrow)$ (abbreviated as $(\mathcal{S}, \mathcal{P}(E))$) a soft set system over U and E . Now we show this system is an information algebra.

Theorem 3.1. The soft set system $(\mathcal{S}, \mathcal{P}(E))$ over U and E is an information algebra.

Proof. Obviously, $\mathcal{P}(E)$ is a lattice composed by the domains of soft sets in \mathcal{S} .

1. Semigroup: Clearly \mathcal{S} is commutative with respect to the operation \sqcap_{ε} . For $A \subseteq E$, the absolute soft set \tilde{A} is the neutral element such that $(F, A) \sqcap_{\varepsilon} \tilde{A} = (F, A)$ for all soft set (F, A) with domain A .

Following we show the associative law holds in the set \mathcal{S} . Let $(F, A), (G, B), (H, C) \in \mathcal{S}$. We write

$$\begin{aligned} (F, A) \sqcap_{\varepsilon} (G, B) &= (Q_1, A \cup B), \\ (G, B) \sqcap_{\varepsilon} (H, C) &= (Q_2, B \cup C), \\ [(F, A) \sqcap_{\varepsilon} (G, B)] \sqcap_{\varepsilon} (H, C) &= (Q_3, A \cup B \cup C), \\ (F, A) \sqcap_{\varepsilon} [(G, B) \sqcap_{\varepsilon} (H, C)] &= (Q_4, A \cup B \cup C). \end{aligned}$$

We need to show that $Q_3 = Q_4$. For any an $e \in A \cup B \cup C$, it can be divided into seven conditions as follows: $e \in (A - B) - C, e \in (B - A) - C, e \in (A \cap B) - C, e \in C - (A \cup B), e \in (A - B) \cap C, e \in (B - A) \cap C$ and $e \in A \cap B \cap C$. Here we take the condition of $e \in (A \cap B) - C$ as an example to illuminate the proof. Assume that $e \in (A \cap B) - C$. Since $(A \cap B) - C = A \cap (B - C)$, we have $e \in A \cap (B - C)$. Then

$$Q_3(e) = Q_1(e) = F(e) \cap G(e),$$

and

$$Q_4(e) = F(e) \cap Q_2(e) = F(e) \cap G(e).$$

So $Q_3(e) = Q_4(e)$. The other conditions are also easy to show. Therefore, the associative law holds.

2. According to these related definitions, the proof of the axioms of labeling, marginalization, transitivity and idempotency are directly.

3. Stability: For $A \in \mathcal{P}(E)$, the neutral element with domain A is the absolute soft set \tilde{A} . Furthermore, if $B \subseteq A$, we have $\tilde{A}^{\downarrow B} = \tilde{B}$. Thus the stability is true.

4. Combination: For $(F, A), (G, B) \in \mathcal{S}$, we need to show

$$((F, A) \sqcap_{\varepsilon} (G, B))^{\downarrow S} = (F, A) \sqcap_{\varepsilon} (G, B)^{\downarrow S \cap B},$$

if $A \subseteq S \subseteq A \cup B$.

In fact, let

$$\begin{aligned}(F, A) \sqcap_\varepsilon (G, B) &= (H, A \cup B), \\ (F, A) \sqcap_\varepsilon (G, B)^{\downarrow^{S \cap B}} &= (H', S).\end{aligned}$$

For all $e \in S$, we have

$$H(e) = H'(e) = \begin{cases} F(e), & \text{if } e \in S \cap (A - B); \\ G(e), & \text{if } e \in S \cap (B - A); \\ F(e) \cap G(e), & \text{if } e \in A \cap B. \end{cases}$$

Then $((F, A) \sqcap_\varepsilon (G, B))^{\downarrow^S} = (F, A) \sqcap_\varepsilon (G, B)^{\downarrow^{S \cap B}}$.

Hence $(\mathcal{S}, \mathcal{P}(E), d, \sqcap_\varepsilon, \downarrow)$ is an information algebra. \square

4. Compact information algebra of soft sets

In general, only “finite” information can be treated in computers. Therefore, a structure called compact information algebra has been proposed by Kohlas. Its main character is that each information can be approximated by these “finite” information with a same domain.

Definition 4.1. ([4]) A system (Φ, Φ_f, D) , where (Φ, D) is an information algebra, the lattice D has a top element,

$$\Phi_f = \bigcup_{x \in D} \Phi_{f,x}$$

where the sets $\Phi_{f,x} \subseteq \Phi_x$ are closed under combination, contain the neutral element $e_x \in \Phi_{f,x}$, and satisfy the following axioms of convergence and density with respect to the ordering relation \leq induced by the operation combination, is called a compact information algebra.

1. Convergency: If $X \subseteq \Phi_{f,x}$ is directed, then the supremum $\vee X$ over Φ exists and $\vee X \in \Phi_x$.

2. Density: For all $\phi \in \Phi_x$,

$$\phi = \bigvee \{\psi \in \Phi_{f,x} : \psi \leq \phi\}.$$

3. Compactness: If $X \subseteq \Phi_{f,x}$ is a directed set, and $\phi \in \Phi_{f,x}$ such that $\phi \leq \vee X$ then there exists a $\psi \in X$ such that $\phi \leq \psi$.

Lemma 4.1. ([4]) If (Φ, Φ_f, D) is a compact information algebra, then $\phi \in \Phi_{f,x}$ if, and only if $\phi \ll \phi$ in set Φ_x .

For convenience, we give an equivalent form for the order relation \leq induced by the operation combination of soft sets.

Proposition 4.1. Let the order relation \leq be induced by the operation combination in the system $(\mathcal{S}, \mathcal{P}(E))$. For two soft sets (F, A) and (G, B) over a common universe U , then $(F, A) \leq (G, B)$ if and only if,

- (i) $A \subseteq B$, and
- (ii) $\forall \varepsilon \in A, G(\varepsilon) \subseteq F(\varepsilon)$.

Proof. We write $(F, A) \sqcap_\varepsilon (G, B) = (H, A \cup B)$.

If $(F, A) \leq (G, B)$, then $(H, A \cup B) = (G, B)$. So $A \cup B = B$, that is, $A \subseteq B$. For all $\varepsilon \in A$, by the definition of the operation \sqcap_ε , we have $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon) = G(\varepsilon)$. Then $G(\varepsilon) \subseteq F(\varepsilon)$ for all $\varepsilon \in A$.

The reverse is also obvious. \square

Proposition 4.2. Let $\{(F_i, A_i) : i \in I\}$ be soft sets over a same universe U . Then

$$\bigvee_{i \in I} (F_i, A_i) = (H, \bigcup_{i \in I} A_i),$$

where $H : \bigcup_{i \in I} A_i \rightarrow \mathcal{P}(U)$ is defined as follows:

$$\forall e \in \bigcup_{i \in I} A_i, \text{ let } J(e) = \{i \in I : e \in A_i\}, H(e) = \bigcap_{i \in J(e)} F_i(e).$$

Proof. Clearly $(H, \bigcup_{i \in I} A_i)$ is an upper bound of $\{(F_i, A_i) : i \in I\}$. Suppose that (G, B) is another upper bound of $\{(F_i, A_i) : i \in I\}$. Thus $\bigcup_{i \in I} A_i \subseteq B$. $\forall e \in \bigcup_{i \in I} A_i, i \in J(e)$, we have $G(e) \subseteq F_i(e)$. Then

$$G(e) \subseteq \bigcap_{i \in J(e)} F_i(e) = H(e).$$

This proves that $(H, \bigcup_{i \in I} A_i) \leq (G, B)$. Thus $\bigvee_{i \in I} (F_i, A_i) = (H, \bigcup_{i \in I} A_i)$. \square

Proposition 4.3. (\mathcal{S}_A, \leq) is a complete lattice. The top element is (\emptyset, A) , and the bottom element is \tilde{A} . Here \mathcal{S}_A is the set of all soft sets with domain A in the system $(\mathcal{S}, \mathcal{P}(E))$.

Proof. For all nonempty subset $\{(F_i, A) : i \in I\} \subseteq \mathcal{S}_A$, by the conclusion of Proposition 4.2, we have

$$\bigvee_{i \in I} (F_i, A) = (H, A) \in \mathcal{S}_A,$$

where $H : A \rightarrow \mathcal{P}(U)$ is defined as $H(e) = \bigcap_{i \in I} F_i(e)$ for all $e \in A$. Moreover, \tilde{A} is the bottom element in the set \mathcal{S}_A . Thus \mathcal{S}_A is a complete lattice.

Lemma 4.2. Let (F, A) be a soft set over a universe U and A be a finite subset of E . Then $(F, A) \ll (F, A)$ in \mathcal{S}_A if, and only if $U - F(e)$ is a finite subset of U for all $e \in A$.

Proof. (1) “if”: Let $\{(G_i, A) : i \in I\}$ be a directed set and $(F, A) \leq \bigvee_{i \in I} (G_i, A)$. We write $\bigvee_{i \in I} (G_i, A) = (G, A)$. $\forall e \in A$, we have $G(e) = \bigcap_{i \in I} G_i(e) \subseteq F(e)$. Then

$$U - F(e) \subseteq U - G(e) = \bigcup_{i \in I} (U - G_i(e)).$$

For any an $x \in U - F(e)$, there is an $i(x) \in I$ such that $x \in U - G_{i(x)}(e)$. Now we get a finite set $\{(G_{i(x)}, A) : x \in U - F(e)\}$, because $U - F(e)$ is finite. By the directness of $\{(G_i, A) : i \in I\}$, there exists an $i^{(e)} \in I$ such that $(G_{i(x)}, A) \leq (G_{i^{(e)}}, A)$ for all $x \in U - F(e)$. Thus $x \in U - G_{i(x)}(e) \subseteq U - G_{i^{(e)}}(e)$. We obtain $U - F(e) \subseteq U - G_{i^{(e)}}(e)$, that is, $G_{i^{(e)}}(e) \subseteq F(e)$.

Since A is a finite set, it implies that $\{(G_{i^{(e)}}, A) : e \in A\}$ is also finite. By the directness of $\{(G_i, A) : i \in I\}$ again, there exists a $j \in I$ such that $(G_{i^{(e)}}, A) \leq (G_j, A)$ for all $e \in A$. We have $G_j(e) \subseteq G_{i^{(e)}}(e) \subseteq F(e)$ for all $e \in A$. This implies $(F, A) \leq (G_j, A)$. Thus $(F, A) \ll (F, A)$ in \mathcal{S}_A .

(2) “only if”: For all $e \in A$, $U - F(e)$ can be represented as the supremum of $\{B_i : i \in I\}$, where $\{B_i : i \in I\}$ is a directed family of all the finite subsets of $U - F(e)$, i.e., $U - F(e) = \bigcup_{i \in I} B_i$. We define a family of soft sets (H_i, A) as follows:

$$H_i(\varepsilon) = \begin{cases} U - B_i, & \text{if } \varepsilon = e; \\ F(\varepsilon), & \text{otherwise.} \end{cases}$$

With respect to the order relation \leq , $\{(H_i, A) : i \in I\}$ is a directed subsets of \mathcal{S}_A . Also we have $(F, A) = \bigvee_{i \in I} (H_i, A)$ by Proposition 4.2. Since $(F, A) \ll (F, A)$ in \mathcal{S}_A , there exists a $k \in I$ such that $(F, A) \leq (H_k, A)$. Hence $U - F(e) \subseteq U - H_k(e) = B_k$. Thus $U - F(e)$ is a finite subset of U . This proves what we have stated. \square

Let $\mathcal{S}_{\mathcal{F}} \subseteq \mathcal{S}$ denote the set of all soft sets with a finite subset of E , i.e.,

$$\mathcal{S}_{\mathcal{F}} = \{(F, A) : (F, A) \text{ is a soft set over } U, \text{ where } A \text{ is a finite subset of } E\}.$$

The symbol $\mathcal{P}_f(E)$ denotes the set of all finite subsets of E .

Let $\mathcal{S}_{f,A} = \{(F, A) : \forall e \in A, U - F(e) \text{ is a finite subset of } U\}$. We denote

$$\mathcal{S}_f = \bigcup_{A \in \mathcal{P}_f(E)} \mathcal{S}_{f,A}.$$

Theorem 4.1. $(\mathcal{S}_{\mathcal{F}}, \mathcal{S}_f, \mathcal{P}_f(E))$ is a compact information algebra.

Proof. First, we have $(\mathcal{S}_{\mathcal{F}}, \mathcal{P}_f(E))$ is an information algebra. It is similar as the proof of Theorem 3.1. By Proposition 4.3, we know (\mathcal{S}_A, \leq) is a complete lattice for all finite subset A of E . Hence the convergency in Definition 4.1 is also true.

By the definition of way-below relation \ll and the conclusion of Lemma 4.2, the compactness is also clear.

Now we need to show the following equation holds for all finite subset $A \subseteq E$,

$$(F, A) = \vee \{(G, A) \in \mathcal{S}_{f,A} : (G, A) \leq (F, A)\}.$$

For all $e \in A$, $U - F(e)$ can be represented as the supremum of $\{B_i : i \in I^{(e)}\}$, i.e., $U - F(e) = \bigcup_{i \in I^{(e)}} B_i$, where $\{B_i : i \in I^{(e)}\}$ is a directed family of all the finite subsets of $U - F(e)$. We define a family of soft sets (F_i, A) as follows:

$$F_i(\varepsilon) = \begin{cases} U - B_i, & \text{if } \varepsilon = e; \\ U, & \text{otherwise.} \end{cases}$$

Let $\mathcal{B} = \{(F_i, A) : i \in I^{(e)}, e \in A\}$. Clearly $\mathcal{B} \subseteq \mathcal{S}_{f,A}$. Meanwhile, for all $(F_i, A) \in \mathcal{B}$, we have $(F_i, A) \leq (F, A)$. In fact, for all $e \in A$, if $i \in I^{(e)}$, we have $F(e) = U - \bigcup_{i \in I^{(e)}} B_i \subseteq U - B_i = F_i(e)$. Otherwise, $F_i(e) = U$. Thus $F(e) \subseteq F_i(e)$ is true. So $(F_i, A) \leq (F, A)$. We write $\bigvee_{(F_i, A) \in \mathcal{B}} (F_i, A) = (H, A)$. For all $d \in A$, we have

$$\begin{aligned} H(d) &= \bigcap_{(F_i, A) \in \mathcal{B}} F_i(d) \\ &= \left(\bigcap_{i \in I^{(d)}} F_i(d) \right) \cap \left(\bigcap_{i \in I^{(e)}, e \in A, e \neq d} F_i(d) \right) \\ &= \left(\bigcap_{i \in I^{(d)}} F_i(d) \right) \cap U \\ &= \bigcap_{i \in I^{(d)}} (U - B_i) \\ &= U - \bigcup_{i \in I^{(d)}} B_i \\ &= F(d). \end{aligned}$$

So we have $F = H$. Therefore,

$$\begin{aligned} (F, A) &= (H, A) \\ &= \bigvee_{(F_i, A) \in \mathcal{B}} (F_i, A) \\ &\leq \vee \{(G, A) \in \mathcal{S}_{f,A} : (G, A) \leq (F, A)\} \\ &\leq (F, A). \end{aligned}$$

Hence

$$(F, A) = \vee \{(G, A) \in \mathcal{S}_{f,A} : (G, A) \leq (F, A)\}.$$

According to the proof above, we obtain that $(\mathcal{S}_{\mathcal{F}}, \mathcal{S}_f, \mathcal{P}_f(E))$ is a compact information algebra. \square

5. Conclusion

In this paper, by defining the operations combination and projection of soft sets, we obtained the structure of information algebras on the family of all soft sets over an initial universe set and a parameter set. Therefore, a soft set system can be subsumed under the specific instances of information algebra systems. We also gave a model of compact information algebra in a soft set system. We have shown the family of all soft sets with a finite parameter subset can form a compact information algebra.

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